```
Clear["Global`*"]
```

It would take a good deal of time studying **NDSolve** to get expert results. The following is a sample of some results, some not bad, some with quite a bit of room for improvement.

Explicit Method

5. Using numbered line (5), p. 937, with h = 1 and k = 0.5, solve the heat problem, numbered lines (1) - (3) on p. 937, to find the temperature at t = 2 in a laterally insulated bar of length 10 ft and initial temperature f(x) = x(1 - 0.1x).

The problem description referred to numbered lines in the text, and I should set those down here. Also, since the problem specifies numbered line (5), I may need that too.

(1) $u_t = u_{xx}$, $0 \le x \le 1$, $t \ge 0$

(2) u[x, 0] = f[x], (Initial condition)

(3) u[0, t] = u[1, t] = 0 , (Boundary conditions)

(5) $u_{i,j+1} = (1 - 2r) u_{ij} + r (u_{i+1,j} + u_{1-1,j})$, $r = \frac{k}{h^2}$

```
Clear["Global`*"]
```

I am redoing this problem after finishing the section. Crank-Nicolson works well and it pretty fast too. One route to Crank-Nicolson comes from the NDSolve Method Plugin Framework tutorial. The wolfram documentation uses Crank-Nicolson as an example for writing a customized plugin for **NDSolve**.

```
Options[CrankNicolson] = {MaxIterations → 5, Tolerance → Automatic};
CrankNicolson /: NDSolve`InitializeMethod[CrankNicolson,
  stepmode_, sd_, rhs_, state_, OptionsPattern[CrankNicolson]] :=
 Module[{prec, rtol, maxit}, maxit = OptionValue[MaxIterations];
  prec = state@"WorkingPrecision";
  rtol = OptionValue[Tolerance];
  If[rtol === Automatic, rtol = 10^(-prec * 3 / 4)];
  CrankNicolson[maxit, rtol]]

CrankNicolson[maxit_, rtol_]["Step"[f_, h_, t0_, x0_, f0_]] :=
  Module[{J, LU, t1 = t0 + h, x1, f1, residual, err,
    done = False, tol = rtol, count = 0}, x1 = x0 + h f0;
   f1 = f[t1, x1];
   x1 = x0 + (h / 2) (f0 + f1);
   J = f["JacobianMatrix"[t1, x1]];
   LU = IdentityMatrix[Length[x1], SparseArray] - (h / 2) J;
   LU = LinearSolve[LU];
   While[(count ≤ maxit) && ! done, f1 = f[t1, x1];
    residual = x1 - x0 - (h / 2) * (f0 + f1);
    err = Norm[residual, Infinity];
    If[err < tol, done = True
      (*else*), x1 = x1 - LU[residual];
     count++;]];
   If[count > maxit, Message[CrankNicolson::cvmit, maxit];
    x1 = $Failed];
   {x1, f1}];

CrankNicolson[___]["StepInput"] = {"F"["T", "X"], "H", "T", "X", "XP"};
CrankNicolson[___]["StepOutput"] = {"X", "XP"};
CrankNicolson[___]["DifferenceOrder"] := 2;
CrankNicolson[___]["StepMode"] := "Fixed";
```

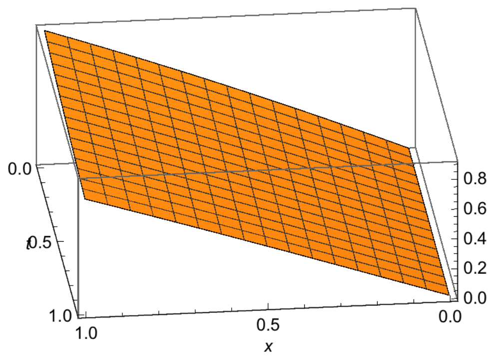
Now comes the part where the problem function is inserted. The execution of this expression is much faster than the generic heat equation which was in the **NDSolve** docs. Raising the accuracy goal above 9 does not result in an improvement in the output. If the working precision is set anywhere except where it is shown below, a warning message appears.

```
cnz = First[
  u /. NDSolve[{D[u[x, t], t] == D[u[x, t], x, x], u[x, 0] == x (1. - 0.1 x),
     u[0, t] == 0, u[10, t] == 0}, u, {x, 0, 2}, {t, 0, 2},
    Method → {"FixedStep", "StepSize" → .001, Method → {CrankNicolson}},
    AccuracyGoal → 15, WorkingPrecision → MachinePrecision]]
```

```
InterpolatingFunction[ Domain: {{0., 10.}, {0., 2.}}  Output: scalar ]
```

I can show a plot.

```
cnn = Plot3D[cnz[x, t], {x, 0, 1},
  {t, 0, 1}, ImageSize → 250, AspectRatio → Automatic,
  ViewPoint → {1000, 9000, 4000.}, AxesLabel → Automatic]
```



The table below shows pretty good agreement with the text answer. The right column is the one for comparison.

```
tableW = TableForm[Table[cnz[h, k], {h, 0, 10}, {k, 0, 2}]]
```

0.	0.	0.
0.9	0.755958	0.667706
1.6	1.41136	1.26027
2.1	1.9016	1.7183
2.4	2.20015	2.00477
2.5	2.30002	2.10191
2.4	2.20015	2.00477
2.1	1.9016	1.7183
1.6	1.41136	1.26027
0.9	0.755958	0.667706
0.	0.	0.

The text answers which are applicable are: 0, 0.6625, 1.25, 1.7125, 2, 2.1, 2, 1.7125, 1.25, 0.6625, 0.

6. Solve the heat problem (1) - (3) by the explicit method with $h = 0.2$ and $k = 0.01$, 8 time steps, when

$$f[x] = \begin{cases} x & 0 \le x < \frac{1}{2} \\ 1 - x & \frac{1}{2} \le x \le 1 \end{cases}$$

Compare with the 3S-values 0.0108, 0.175 for $t = 0.08$, $x = 0.2$, 0.4 obtained from the series (2 terms) in section 12.5.

7. The accuracy of the explicit method depends on r $\left(\le \frac{1}{2}\right)$. Illustrate this for problem 6, choosing $r = \frac{1}{2}$ (and $h = 0.2$ as before). Do 4 steps. Compare the values for $t = 0.04$ and 0.08 with the 3S-values in problem 6, which are 0.156, 0.254 ($t = 0.04$), 0.105, 0.170 ($t = 0.08$).

```
Clear["Global`*"]
```

From numbered line (5) above I have

$$r = \frac{k}{h^2};$$

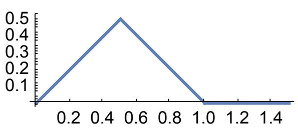
so

```
k = .5 (0.2)^2
0.02
```

$$f[x\_] = \begin{cases} x & 0 \le x < \frac{1}{2} \\ 1 - x & \frac{1}{2} \le x \le 1 \end{cases}$$

$$\begin{cases} x & 0 \le x < \frac{1}{2} \\ 1 - x & \frac{1}{2} \le x \le 1 \\ 0 & \text{True} \end{cases}$$

```
Plot[f[x], {x, 0, 1.5}, AspectRatio → Automatic, ImageSize → 150]
```



I was able to use WolframDocumentation, an article called *tutorial/NDSolveWhenEvents* which has some good info. I guess it sometimes helps to write a function that incorporates the **Piecewise** structure, but which can be called by a single symbol.

```
g1[x_, t_] = x;
g2[x_, t_] = 1 - x;
g[x_, t_] = If[x < 0.5, g1[x, t], g2[x, t]];
```

Though the tutorial did not tell me to do so, using Dirichlet conditions occurred to me as one way to do it. The try below shows what it looked like. However, the next cell down, bsol2, shows a better way for this problem, and bsol is only here for historical interest.

```
bsol = First[u /. NDSolve[{D[u[x, t], t] == D[u[x, t], x, x],
      DirichletCondition[u[x, t] == g[x, t], t == 0 && 0 ≤ x < 0.5],
      DirichletCondition[u[x, t] == 0, x == 0 && t == 0],
      DirichletCondition[u[x, t] == g[x, t], x ≥ 0.5 && t ≥ 0]}, u,
     {x, 0, 1}, {t, 0, 1}, PrecisionGoal → 12, AccuracyGoal → 12,
     WorkingPrecision → 15, MaxStepSize -> 0.002]]
```

NDSolve::femcscd : The PDE is convection dominated and the result may not be stable. Adding artificial diffusion may help. ≫

```
InterpolatingFunction[ Domain: {{0., 1.}, {0., 1.}}  Output: scalar ]
```

After surfing around on a day when StackExchange was down, I found an old post *http://forums.wolfram.com/mathgroup/archive/2008/Sep/msg00458.html* from 2008 that showed putting Piecewise right into the **NDSolve** expression without apologies. So I tried it as below.

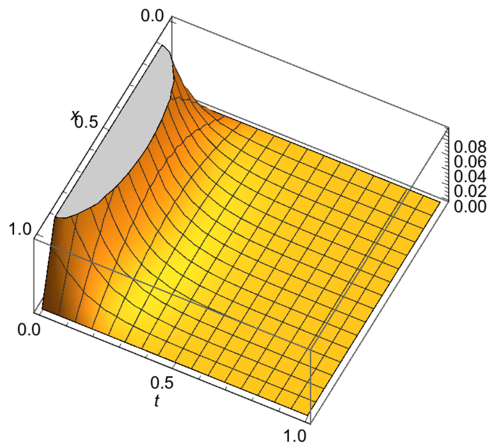
```
bsol2 = First[u /. NDSolve[{D[u[x, t], t] == D[u[x, t], x, x],
      u[x, 0] == Piecewise[{{x, 0 ≤ x < 0.5}, {1 - x, 0.5 ≤ x ≤ 1}}],
      u[0, t] == 0, u[1, t] == 0}, u, {x, 0, 1}, {t, 0, 1}, AccuracyGoal → 12,
     WorkingPrecision → MachinePrecision, MaxStepSize -> 0.1]]
```

NDSolve::mxsst:
Using maximum number of grid points 10000 allowed by the MaxPoints or MinStepSize options for independent variable x. >>

```
InterpolatingFunction[ Domain: {{0., 1.}, {0., 1.}}
                       Output: scalar ]
```

And the **First** formulation allows me to conveniently plot it. This is a nicer plot than the one for the expression with g[x].

```
wiwp = Plot3D[bsol2[x, t], {x, 0, 1},
  {t, 0, 1}, ImageSize → 250, AspectRatio → Automatic,
  ViewPoint → {4000, 2000, 6000.}, AxesLabel → Automatic]
```



And I can build a table with solution values. As the problem specifies, the h step size is 0.2 and the k step size is 0.02. The table values are difficult to interpret.

```
TableForm[Table[bsol2[h, k], {h, 0, 1, 0.4}, {k, 0, 0.08, 0.02}]]
```

0.	0.	0.	0.	6.77626×10^{-21}
0.4	0.320886	0.260491	0.213326	0.17503
0.2	0.188294	0.15928	0.131562	0.108129

The text answers for t=0.04 have 0.156 and 0.254, for t=0.08 the text lists 0.105 and 0.170. If the individual sets were flipped around, or the above table (third column) viewed from bottom going up, they would roughly agree. Assuming the explanation just made is acceptable, the results are not too bad. But maybe I need to check the performance of FEM on the same job. (This check I just made, and, the result being just the same as the above, I deleted it.)

8. In a laterally insulated bar of length 1 let the initial temperature be f[x] = x if 0 ≤ x < 0.5, f[x] = 1 - x if 0.5 ≤ x ≤ 1. Let (1) and (3) hold. Apply the explicit method with h = 0.2, k = 0.01, 5 steps. Can you expect the solution to satisfy u[x,t] = u[1 - x, t] for all t?

9. Solve problem 8 with f[x] = x if 0 ≤ x ≤ 0.2, f[x] = 0.25(1 - x) if 0.2 < x ≤ 1, the other data being as before.

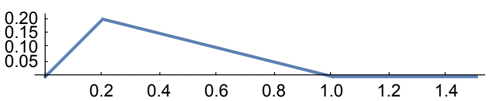
Here I use forthright identification of the function f[x] as piece-wise and get away with it, without stating any certain method for use with **NDSolve**. That is, no special modifications or adaptations are necessary.

```
Clear["Global`*"]
```

$$f[x\_] = \begin{cases} x & 0 \le x < 0.2 \\ 0.25\,(1 - x) & 0.2 \le x \le 1 \end{cases}$$

$$\begin{cases} x & 0 \le x < 0.2 \\ 0.25\,(1 - x) & 0.2 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

```
Plot[f[x], {x, 0, 1.5}, AspectRatio → Automatic, ImageSize → 250]
```



As before, I will not actually use the g[x] function here, merely show what they would look like for this problem.

```
g1[x_, t_] = x;
g2[x_, t_] = .25 (1 - x);
g[x_, t_] = If[x < 0.2, g1[x, t], g2[x, t]];
```

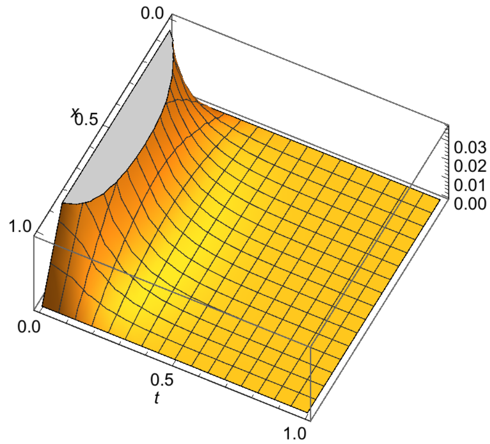
Again, below, the addition of enhancing factors in the **NDSolve** expression is cheap. However, the difference in the solution between step size 0.1 and 0.001 is in the 5th decimal place, so it is not really worth putting in the smaller step size. Mathematica gives me one warning message, but then goes ahead and does the calculation, without freezing.

```
bsol2 = First[u /. NDSolve[{D[u[x, t], t] == D[u[x, t], x, x],
     u[x, 0] == Piecewise[{{x, 0 ≤ x < 0.2}, {0.25 (1 - x), 0.2 ≤ x ≤ 1}}],
     u[0, t] == 0, u[1, t] == 0}, u, {x, 0, 1}, {t, 0, 1}, AccuracyGoal → 12,
    WorkingPrecision → MachinePrecision, MaxStepSize -> 0.1]]
```

NDSolve::mxsst :
Using maximum number of grid points 10000 allowed by the MaxPoints or MinStepSize options for independent variable x. ≫

InterpolatingFunction[Domain: {{0., 1.}, {0., 1.}} Output: scalar]

```
wiwp = Plot3D[bsol2[x, t], {x, 0, 1},
  {t, 0, 1}, ImageSize → 250, AspectRatio → Automatic,
  ViewPoint → {4000, 2000, 6000.}, AxesLabel → Automatic]
```




```
TableForm[Table[bsol2[h, k], {h, 0, 1.0, 0.2}, {k, 0, 0.05, 0.01}]]
```

0.	0.	0.	0.	0.	0.
0.2	0.129594	0.102395	0.0844395	0.0715155	0.061685
0.15	0.143718	0.129259	0.114964	0.102247	0.0911765
0.1	0.0998782	0.097881	0.0934618	0.0876805	0.0813476
0.05	0.0499996	0.0499048	0.049288	0.0478799	0.0457676
0.	0.	0.	0.	0.	0.

The values in the table are similar to the answer in the text (last column). For info, the text answer is 0, 0.06279, 0.09336, 0.08364, 0.04707, 0.

Crank-Nicolson method

11. Solve problem 9 by (9) with h = 0.2, 2 steps. Compare with exact values obtained from the series in section 12.5 (2 terms) with suitable coefficients.

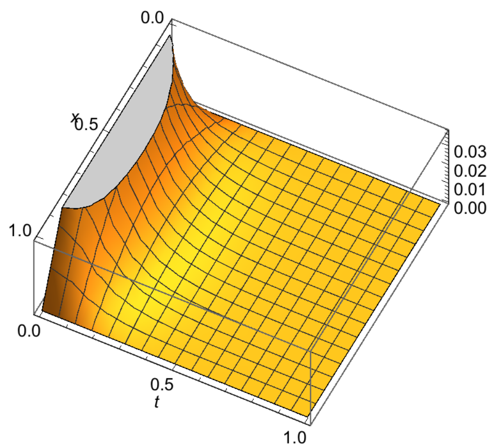
There are some good, recent notebooks on numerical Mathematica at *https://sites.google.com/site/chemengwithmathematica/home/numerical-methods.* The Crank-Nicolson is under Numerical Methods for PDEs, and some code used later on is taken from that notebook. BGHiggins is the author. The code was not well adapted to the present problem, so I removed it. But it is used in problem 15. In its place is a FEM version which I believe works fairly well.

```
Clear["Global`*"]
```

```
uif = First[u /. NDSolve[{D[u[x, t], t] == D[u[x, t], x, x],
     u[x, 0] == Piecewise[{{x, 0 ≤ x < 0.2}, {0.25 (1 - x), 0.2 ≤ x ≤ 1}}],
     u[0, t] == u[1, t] == 0}, u, {x, 0, 1}, {t, 0, 1},
    Method → {"MethodOfLines", "SpatialDiscretization" →
       {"FiniteElement", "MeshOptions" → {"MaxCellMeasure" → 0.001}}},
    AccuracyGoal → 12, WorkingPrecision → MachinePrecision,
    MaxStepFraction → 0.001]]
```

```
InterpolatingFunction[ Domain: {{0., 1.}, {0., 1.}} Output: scalar ]
```

```
Plot3D[uif[x, t], {x, 0, 1}, {t, 0, 1}, AxesLabel → Automatic,
 ImageSize → 250, ViewPoint → {4000, 2000, 6000.}]
```




```
TableForm[Table[uif[h, k], {h, 0, 1.0, 0.2}, {k, 0.03, 0.08, 0.01}]]
```

0.	0.	0.	0.	0.	0.
0.0844303	0.0715163	0.0616857	0.0538905	0.0475197	0.0421898
0.114969	0.102245	0.0911758	0.0815588	0.0731626	0.0657838
0.0934634	0.0876817	0.0813474	0.0749336	0.0686977	0.0627752
0.0492862	0.0478801	0.0457683	0.0431678	0.0402949	0.0373216
0.	0.	0.	0.	0.	2.71051 × 1

I think the redirections on this problem may have messed things up for the designation of the desired answer. The answer in the text for exact values for "Step 2" is as follows: 0, 0.0422, 0.0658, 0.0628, 0.0373, 0. These numbers should be compared with the last column above, with which their resemblance is very suggestive.

13 - 15 Solve (1) - (3) by Crank-Nicolson with r = 1 (5 steps), where

13. $f[x\_]= \begin{cases} 5x & 0 \le x < 0.25 \\ 1.25\,(1-x) & 0.25 \le x \le 1 \end{cases}$, h = 0.2

Since k = r h^2, k = 0.04.

```
Clear["Global`*"]
```

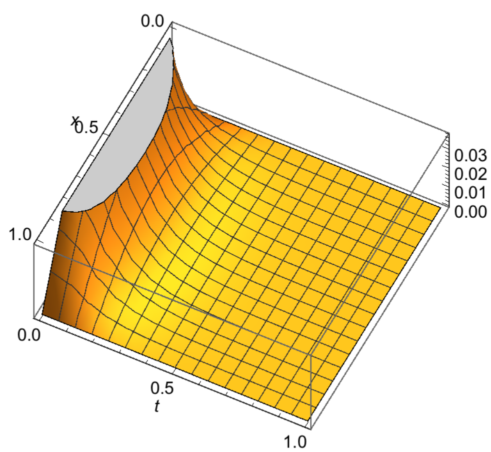
The Crank-Nicolson method is one I haven't been able to get to work well with **Piecewise** functions. I substitute a FEM method approach here. It doesn’t do too badly. The current problem’s equation for f[x] allows setting the MaxCellMeasure quite small.

```
uif = First[u /. NDSolve[{D[u[x, t], t] == D[u[x, t], x, x],
     u[x, 0] == Piecewise[{{5 x, 0 ≤ x < 0.25}, {1.25 (1 - x), 0.25 ≤ x ≤ 1}}],
     u[0, t] == u[1, t] == 0}, u, {x, 0, 1}, {t, 0, 1},
    Method → {"MethodOfLines", "SpatialDiscretization" →
       {"FiniteElement", "MeshOptions" → {"MaxCellMeasure" → 0.001}}},
    AccuracyGoal → 12, WorkingPrecision → MachinePrecision,
    MaxStepFraction → 0.001]]
```

```
InterpolatingFunction[ Domain {{0., 1.}, {0., 1.}}
                       Output scalar ]
```

The plot doesn’t look too ragged.

```
Plot3D[uif[x, t], {x, 0, 1}, {t, 0, 1}, AxesLabel → Automatic,
 ImageSize → 250, ViewPoint → {4000, 2000, 6000.}]
```




```
TableForm[Table[uif[h, k], {h, 0.2, 0.8, 0.2}, {k, 0, 0.05, 0.01}]]
```

1.	0.669005	0.525685	0.432009	0.365051	0.31432
0.75	0.729531	0.657158	0.584413	0.519525	0.46307
0.5	0.500162	0.492285	0.471403	0.442987	0.411421
0.25	0.250005	0.249805	0.247297	0.240828	0.230672

The table above is not in excellent agreement with the text answer, with some values low, others high, in the 5th column, the one sought. The text answers there are 0.3301, 0.5706, 0.4522, 0.2380.

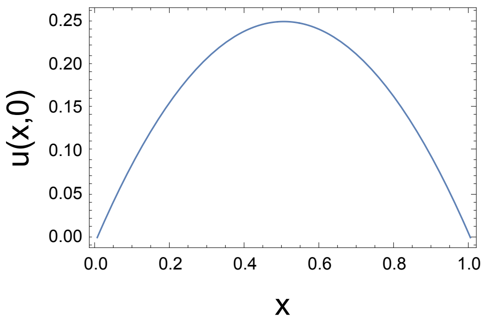
15. f[x_]=x(1-x) , h=0.2

This problem, involving as it does a continuous function for f[x], gives me an opportunity to authenticate the accuracy of the CrankNicolson, using Professor Higgins’s code.

```
Clear["Global`*"]
```

```
LX = 1;
NP = 80;
Δx = LX / (NP - 1) // N;
GridRules = Table[x[i] → (i - 1) Δx, {i, 1, NP}];
f[x_] := x (1 - x)
```

```
Plot[f[x], {x, 0, 1}, PlotStyle → Thickness[0.004], Frame → True,
 FrameLabel → {Style["x", 16], Style["u(x,0)", 16]}, ImageSize → 250]
```




```
𝕂 = 1;
Δt = Δx²;
λ = 𝕂 Δt / Δx²;
```

```
eqnTemplate := λ u2[i + 1] - 2 (1 + λ) u2[i] + λ u2[i - 1] ==
   -λ u1[i + 1] - 2 (1 - λ) u1[i] - λ u1[i - 1]
FDEqns = Table[eqnTemplate, {i, 2, NP - 1}] /.
    {u2[1] → 0, u2[NP] → 0, u1[1] → 0, u1[NP] → 0};
ICvar = Union[Cases[FDEqns, u1[_], ∞]];
var = Union[Cases[FDEqns, u2[_], ∞]];
```

Below I reduced NT from 500 to 250 in order to access the plot for t = 0.04.

```
For[NT = 250;
 ICond = Table[u1[i] -> f[x] /. x → x[i], {i, 2, NP - 1}] /. GridRules;
 j = 1, j < NT, {b, A} = CoefficientArrays[FDEqns /. ICond, var];
 sol = LinearSolve[A, -b];
 ICond = Thread[ICvar → sol];
 j++]
```

According to the text answer, the solution for the first step of h = 0.2 is 0.1018, at t = 0.04, and that looks pretty close to the plot value. To be more exact, the two closest values in hvc below bracket 0.1018, are: {0.189873,0.0980386},{0.202532,0.103677}, from which the interpolated function value for x = 0.2 would be 0.1025, reasonably close.

```
hvc = Transpose[
   {Table[x[i], {i, 1, NP}] /. GridRules, Append[Prepend[sol, 0], 0]}];
```

Now I'm going to try to check the solution using a series solution, which is also provided by the same CN notebook of Higgins. If I try to use the continuous function f[x]=x, then the appropriate parameters for this problem (doing just f1) are

$k = 1$, $L = 0.2$, $f(x) = x$;

Hence, the mathematical statement of the example problem solved by the finite difference method is

$$\frac{\partial u}{\partial t} = \mathbb{K}\,\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad t > 0$$
$$\text{IC}: \quad u(x, 0) = f(x) \tag{1}$$
$$\text{BC1}: \quad u(0, t) = 0$$
$$\text{BC2}: \quad u(L, t) = 0$$

The solution found by separation of variables is

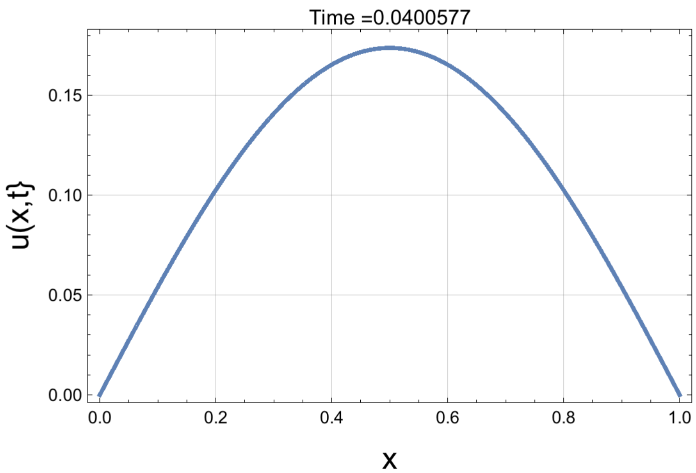
$$u(x, t) = \sum_{n=1}^{\infty} b_n \,\mathrm{Sin}(n\pi x / L)\, e^{-\mathbb{K}\,(n\pi/L)^2\, t}$$

where the Fourier coefficients are given by

$$b_n = \frac{2}{L}\int_0^L f(x)\,\mathrm{Sin}(n\pi x/L)\,dx$$

The Fourier coefficients and the resulting plot are given below.

```
fd = ListPlot[Transpose[
    {Table[x[i], {i, 1, NP}] /. GridRules, Append[Prepend[sol, 0], 0]}],
   Joined → True, Frame → True, PlotStyle → Thickness[0.007],
   FrameLabel → {Style["x", 16], Style["u(x,t}", 16]},
   PlotLabel → "Time =" <> ToString[NT Δt], GridLines → Automatic]
```

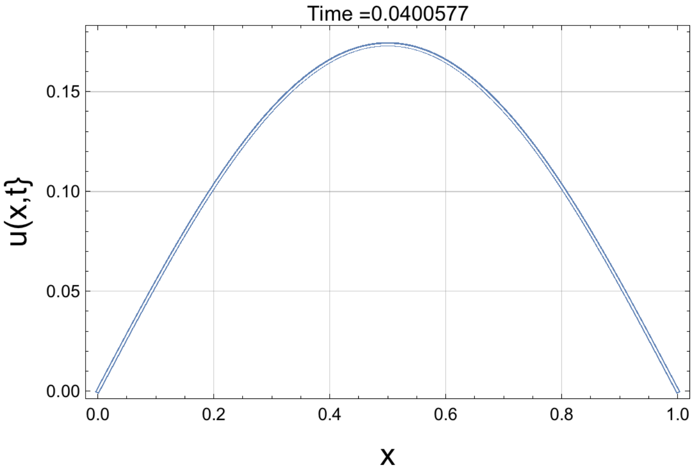


And now the series plot.

```
bb[n_] := bb[n] = 2 NIntegrate[ f[x] Sin[n Pi x], {x, 0, 1}];
Table[bb[n], {n, 1, 30}];
𝕂 = 1;
F[x_, t_, n_] := Sum[bb[j] Sin[j Pi x] Exp[-𝕂 j^2 Pi^2 t],
                   {j, 1, n}];
fs = Plot[F[x, NT Δt, 20], {x, 0, 1}, PlotRange → Automatic,
    Frame → True, PlotStyle → {White, Thickness[0.003]},
    FrameLabel → {Style["x", 16], Style["u(x,t}", 16]},
    PlotLabel → "Time =" <> ToString[NT Δt]];
```

When the series version plot is overlaid on top of the solution version plot, a close agreement (though not perfect) can be seen.

```
Show[fd, fs]
```



Following up with more solutions to the current problem. Below, I dust off the method mentioned in problem 9 as an alternative route to Crank-Nicolson.

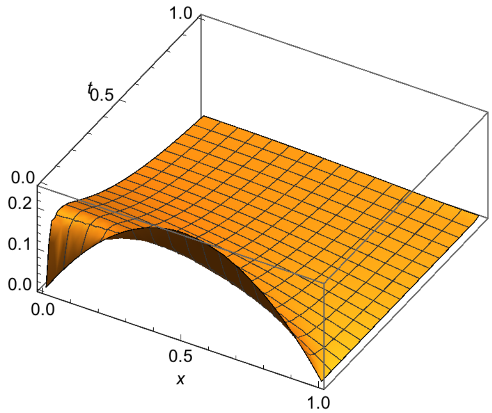
```
Clear["Global`*"]

cnsol = First[u /. NDSolve[{D[u[x, t], t] == D[u[x, t], x, x],
      DirichletCondition[u[x, t] == x (1 - x), t == 0 && 0 ≤ x < 1],
      DirichletCondition[u[x, t] == 0, x == 0 && t == 0],
      DirichletCondition[u[x, t] == 0, x == 1 && 0 ≤ t < 1]}, u, {x, 0, 1},
     {t, 0, 1}, Method → {"DoubleStep", Method -> CrankNicolson},
     PrecisionGoal → 16, AccuracyGoal → 16,
     WorkingPrecision → 20, MaxStepSize -> 0.001]]
```

NDSolve::femcscd: The PDE is convection dominated and the result may not be stable. Adding artificial diffusion may help. ≫

```
InterpolatingFunction[ Domain: {{0., 1.}, {0., 1.}}
                       Output: scalar ]
```

```
pc = Plot3D[cnsol[x, t], {x, 0, 1},
  {t, 0, 1}, ImageSize → 250, AspectRatio → Automatic,
  ViewPoint → {4000, -7000, 6000.}, AxesLabel → Automatic]
```



The third column of the table below would be the one I would try to match with text answer. The second and third entries are not terrible, but not great either.

```
TableForm[Table[cnsol[h, k], {h, 0, 1.0, 0.3}, {k, 0, 0.06, 0.02}]]
```

0.	0.112802	0.153099	0.15324
0.21	0.184225	0.170901	0.165557
0.24	0.201446	0.168982	0.144972
0.09	0.0679211	0.0536557	0.0452542

I'm not sure if that last one really executed the Crank-Nicolson method, though it was called for. A firmer basis for believing CN is being seen in action is contained in the next blocks.

```
Clear["Global`*"]
```

```
Options[CrankNicolson] = {MaxIterations → 5, Tolerance → Automatic};
CrankNicolson /: NDSolve`InitializeMethod[CrankNicolson,
   stepmode_, sd_, rhs_, state_, OptionsPattern[CrankNicolson]] :=
 Module[{prec, rtol, maxit}, maxit = OptionValue[MaxIterations];
   prec = state@"WorkingPrecision";
   rtol = OptionValue[Tolerance];
   If[rtol === Automatic, rtol = 10^(-prec * 3 / 4)];
   CrankNicolson[maxit, rtol]]
```

```
CrankNicolson[maxit_, rtol_]["Step"[f_, h_, t0_, x0_, f0_]] :=
  Module[{J, LU, t1 = t0 + h, x1, f1, residual, err,
     done = False, tol = rtol, count = 0}, x1 = x0 + h f0;
    f1 = f[t1, x1];
    x1 = x0 + (h / 2) (f0 + f1);
    J = f["JacobianMatrix"[t1, x1]];
    LU = IdentityMatrix[Length[x1], SparseArray] - (h / 2) J;
    LU = LinearSolve[LU];
    While[(count ≤ maxit) && ! done, f1 = f[t1, x1];
     residual = x1 - x0 - (h / 2) * (f0 + f1);
     err = Norm[residual, Infinity];
     If[err < tol, done = True
      (*else*), x1 = x1 - LU[residual];
      count++;]];
    If[count > maxit, Message[CrankNicolson::cvmit, maxit];
     x1 = $Failed];
    {x1, f1}];
```

```
CrankNicolson[___]["StepInput"] = {"F"["T", "X"], "H", "T", "X", "XP"};
CrankNicolson[___]["StepOutput"] = {"X", "XP"};
CrankNicolson[___]["DifferenceOrder"] := 2;
CrankNicolson[___]["StepMode"] := "Fixed";
```

Here is included the test sample from the documentation. Note below that the step size in the example function is 0.1. I will change this later.

```
NDSolve[{x''[t] + x[t] == 0, x[0] == 1, x'[0] == 0}, x,
 {t, 0, 2 π}, Method → {"FixedStep", "StepSize" → .1, Method →
    {CrankNicolson, MaxIterations → 1, Tolerance → $MachineEpsilon}}]
```

```
{{x → InterpolatingFunction[ Domain {{0., 6.28}}  Output scalar ]}}
```

```
TableForm[Table[cns[h, k], {h, 0, 1.0, 0.3}, {k, 0, 0.06, 0.02}]];
```

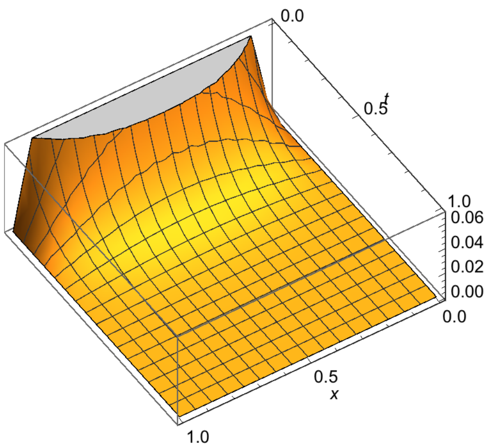
Now that I have tried the example and all seems to be working, I will plug in the problem function. The lines shown reflect some tinkering, by adding the enhancing options and reducing the step size. Smaller step size was very cheap, almost seemed free.

```
cnz =
 First[u /. NDSolve[{D[u[x, t], t] == D[u[x, t], x, x], u[x, 0] == x (1 - x),
     u[0, t] == 0, u[1, t] == 0}, u, {x, 0, 1}, {t, 0, 1},
    Method → {"FixedStep", "StepSize" → .001, Method → {CrankNicolson}},
    PrecisionGoal → 9, AccuracyGoal → 9, WorkingPrecision → 12]]
```

```
InterpolatingFunction[ Domain: {{0, 1.00000000000}, {0, 1.}}
                       Output: scalar ]
```

The plot I get is much smoother than some previous ones. Oddly, it looks symmetrical across the x interval lines, with x = 0.5 as the fulcrum.

```
cnn = Plot3D[cnz[x, t], {x, 0, 1},
  {t, 0, 1}, ImageSize → 250, AspectRatio → Automatic,
  ViewPoint → {4000, 7000, 6000.}, AxesLabel → Automatic]
```



In the table, the h values go down vertically, the k values go across horizontally. The third column is t=0.04, and it looks more or less acceptable.

```
tableW = TableForm[Table[cnz[h, k], {h, 0, 1.0, 0.2}, {k, 0, 0.06, 0.02}]]
```

0.	0.	0.	0.
0.16	0.126025	0.102449	0.0839276
0.24	0.200478	0.165185	0.135699
0.24	0.200478	0.165185	0.135699
0.16	0.126025	0.102449	0.0839276
0.	0.	0.	0.

The grid below shows reasonable agreement with the text answer.

```
Grid[{{"tableW", "text ans"},
  {0, 0}, {0.102449, 0.1018}, {0.165185, 0.1673},
  {0.165185, 0.1673}, {0.102449, 0.1018}, {0, 0}}, Frame → All]
```

tableW	text ans
0	0
0.102449	0.1018
0.165185	0.1673
0.165185	0.1673
0.102449	0.1018
0	0